Midterm Exam : Waves & Optics

11 December 2017, 9:00-11:00

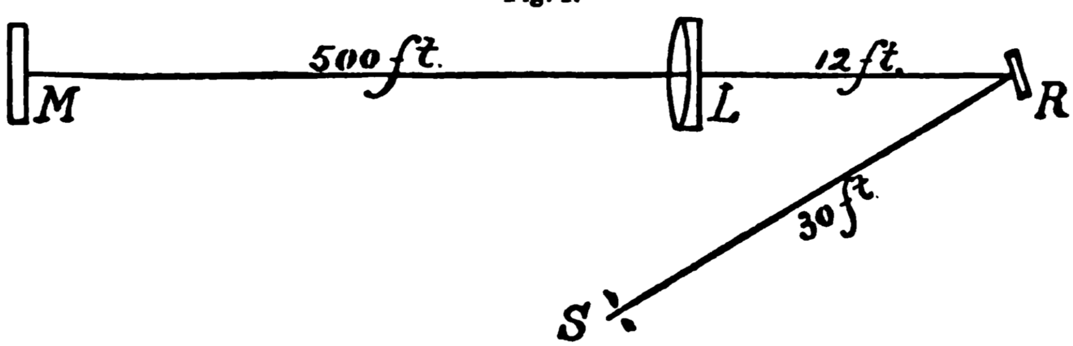
- Put your name and student number on each answer sheet.
- Answer all questions short and to the point, but complete; write legible.
- Final point grade = 9(total number of points/30) + 1

1. **Speed of Light (9 points)**
   consider the paper *Experimental Determination of the Velocity of Light* by Michelson.

   a) In the experiment, the velocity of light *in air* is measured. Briefly explain why the addition "*in air*" is important. **(3 points)**
   The measured velocity is $v = c/n$, with n the refractive index. If n is not known *a priori*, it is not possible to extract c.

   b) Fig. 1 of the paper (and the figure below) gives a schematic overview of the rotating mirror setup used to measure the velocity of light (see below for an short explanation). If you want to accurately measure the velocity of light in vacuum, which of the components M, L, R and S **must** be placed inside vacuum to avoid errors and which **could** be placed outside? Briefly motivate your answer. **(3 points)** .

   Fig. 1.



   Ideally the vacuum tank covers the whole setup. Minimally M, L and R should be in the tank, as MR is the distance over which the light travels while the mirror rotates. So RS does not affect the vecocity measurements, and thus S could be placed outside of the vacuum.

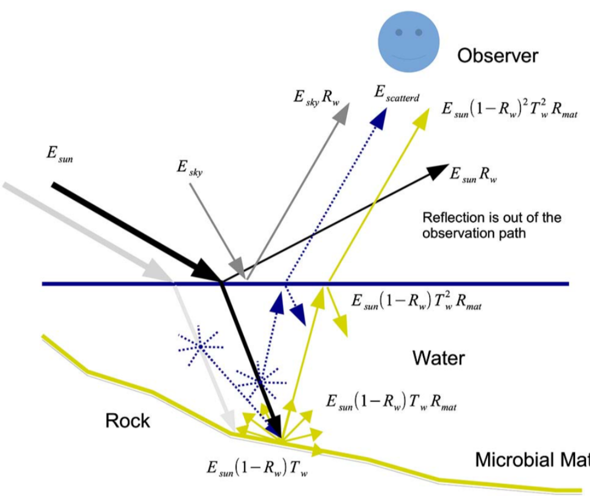
   c) Instead of using sunlight, you repeat the velocity measurement using a red, green, and blue laser. In the measurements in air you notice that the results obtained for the three lasers differ from each other, whereas that is not the case for vacuum. Explain which physics phenomenon in the gases is responsible for this and for which of the three colors you expect to measure the lowest velocity. **(3 points)**
   Dispersion, i.e. frequency or wavelength dependent index of refraction. Normal dispersion, which is typical for visible light, leads to a larger refractive index for larger frequencies. Blue light has the highest frequency, and thus the lowest velocity.

* Sunlight is admitted to the setup through a slit S and aimed at a rotating mirror R. This mirror is placed in the focal point of a lens L, which creates a parallel beam which is reflected from a mirror M. The reflected beam is focused onto the rotating mirror and reflected towards the slit. For a known revolution rate and fixed speed of light the reflected beam will be (slightly) rotated with respect to the incoming beam. By observing the displacement of the reflected beam with respect to the slit the velocity of light can be deduced.

2. **Thermal Pools (12 points)**
refers to paper *Colors of thermal pools at Yellowstone National Park* by Nugent, Shaw, and Vollmer.

   a) Why is it that suspended particles in the water cause a blue color, particularly for deep water? **(3 points)**
   This is due to Rayleigh scatterings, which is stronger for short wavelengths ($1/\lambda^4$ dependence). So blue light will exhibit more (back)scattering than red light.

   b) Consider fig. 2 from the paper (reproduced below; left). Direct sunlight E_{sun} (coming in from the left) is unpolarized. Use figure 4.49 in Hecht (also reproduced; right) to make a rough estimate of the degree of polarization of the reflected light for the configuration shown in the left figure (*e.g.* not at all, somewhat, ..., very much, complete). You may assume that water and glass have about the same index of refraction. Which equation in the book would you need for a full calculation? **(3 points)**



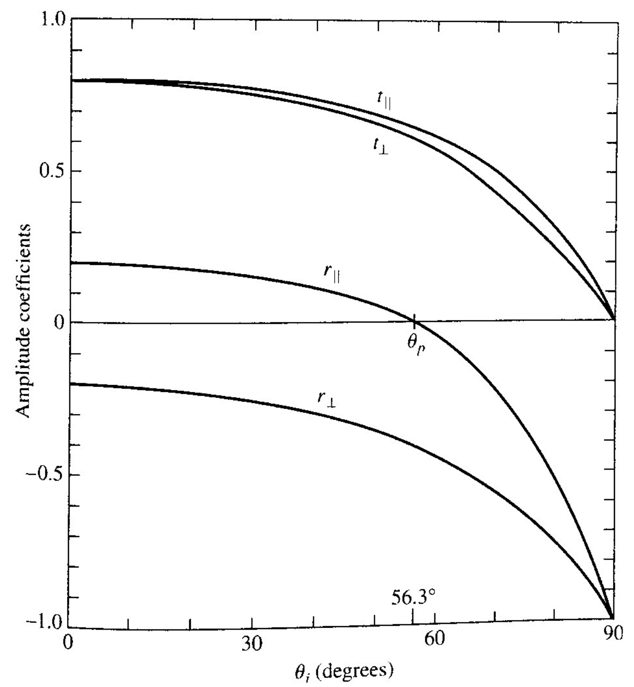


Figure 4.49 The amplitude coefficients of reflection and transmission as a function of incident angle. These correspond to external reflection $n_t > n_i$ at an air–glass interface ($n_{ti} = 1.5$).

   In the left figure it can be read that the beam comes in with an angle of about 60 degrees with respect to the normal (so 30 degrees wrt the horizontal is wrong). At this angle of incidence, the reflective coefficient for the parallel component is about zero, whereas that for the perpendicular component is about -0.3 to -0.4, as can be read from the right figure. Because of the zero reflection of the parallel component, the reflected light is almost fully polarized. For a full calculation, you would need the Fresnel equations, e.g eqn. 4.42–4.45 in Hecht.

   c) Consider equation 1 of the paper:

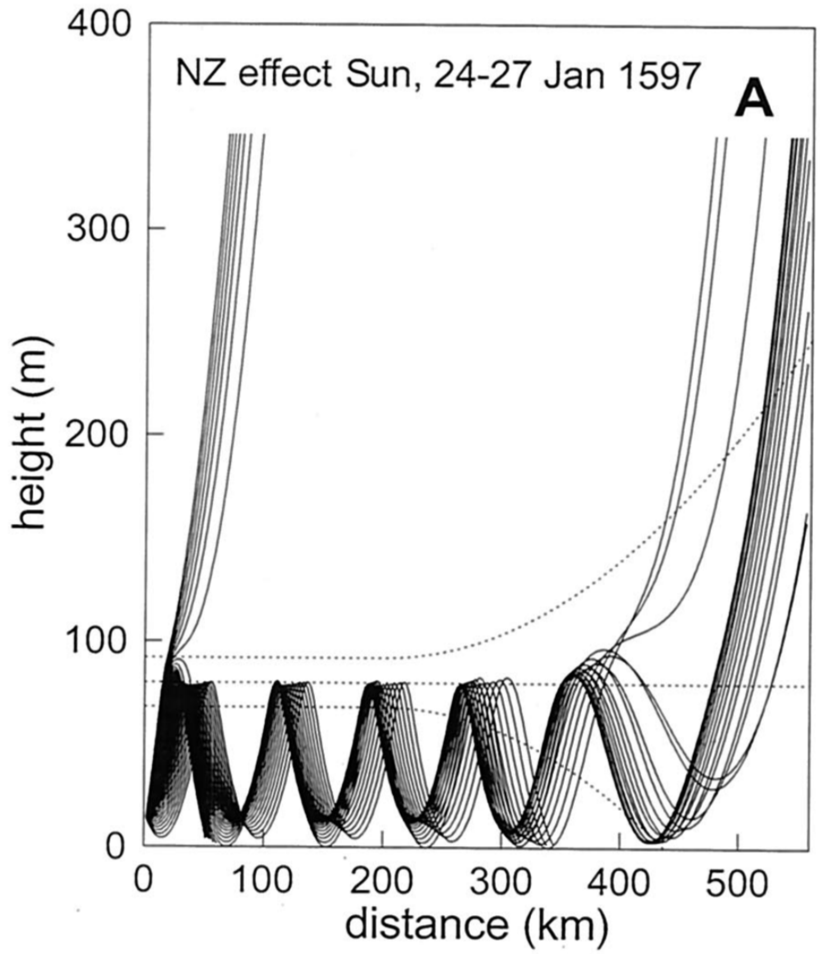
$$E(\lambda) = E_{sky}(\lambda)R_w + E_{sun}(\lambda)(1-R_w)^2R_{mat}T_w^2 + E_{scattered}(\lambda) \tag{1}$$

Why does T_w depend on the depth of the water, and the two R's not? Explain why T_w and the R's may also be wavelength dependent. **(3 points)**
The two R's refer to reflection coefficients. Reflection is a phenomenon that takes place at the interface between two media, in this case between air and water (R_W) or between water and the bacerial mat (R_{mat}) and is hence not depth dependent. The transmisstion through water is indicated by T. As there is a certain degree of absorption per unit length, the total absorption and thus transmission depends on the amount of water to be transversed, and hence the depth of the water, as well as the angle of the rays with respect to the water surface.

d) Sunlight ray enters the water and propagates in a different direction below the water surface as above it (indicated by the thick arrows in the figure). Explain how you can use the geometrical properties of the bending ray to calculate the refractive index of water. Copy the figure with all relevant ingredients on your answer form and indicated how the various parameters in your explanation are defined. **(3 points)**
The connection between the geometrical properties of the rays, i.e. the angle of the incoming and transmitted ray wrt to the surface, are related through Snell's law, which also includes the two indices of refraction: $n_i \sin\theta_i = n_t \sin\theta_t$. Here $n_i \simeq 1$ is the index of refraction of air, θ_i is the angle of incidence with respect to the normal on the water surface. The refractive index of water is n_t, and the transmission angle θ_t is again measured wrt to the normal on the water-air interface.

3. **Novaya Zemblya Effect (9 points)**
refers to paper *Gerrit de Veer's true and perfect description of the Novaya Zemlya effect, 24–27 January 1597* by van der Werf,Könthe, Lehn, Steenhuisen, and Davidson.

a) Fig. 3 of the paper (reproduced below) shows the ray-tracing from the observer's position (left) for the Sun (solid curves) to explain the Novaya Zemblya effect, which is attributed to the bending of light due to a temperature inversion. Isotherms are shown as dashed curves. Why do the rays bend back up near the surface of the earth? **(3 points)**



Because rays near the minimal height are actually straight lines parallel to the surface of the earth. As the earth "sinks" away the inversion layer, which is at fixed height relative to the surface, comes closer and reflection occurs bending the ray back towards the surface of the earth.

b) The NZ effect can be interpreted as a form of total interal reflection. Under which condition does total internal reflection occur? Use the forced oscillator model for the index of refraction and the ideal gas law to argue why temperature *inversion* is necessary to have an NZ effect. **(3 points)**
TIR: $sin(\theta_{in}) = n_{small}/n_{large}$. Light travels in medium with higher n than it's surroundings. IGL: higher temperature implies lower density and thus smaller refractive index (closer to 1). So the colder layer of air behaves as a "fiber" with a high n, whereas the warmer layer

on top of it has a low n.

c) If the temperature inversion would be higher up, the NZ effect would be more dramatic, and the sun would rise above the horizon even earlier. Argue whether this is true or not. Is there a maximum height for the inversion? Does that height depend on the magnitude of the temperature jump? (see e.g. fig.2 in the paper) **(3 points)**
IF the back-reflection would occur, then a higher inversion layer would indeed lead to a more dramatic effect because the width of the undulating pattern in the figure would become larger. However, the higher the layer is, the more perpendicular the rays hit it. So with an insufficient change in the refractive index, and thus insufficient change in temperature, internal reflection would no longer occur.